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Economic Design of $\overline{X}$ Control Charts for Monitoring a First Order Autoregressive Process

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Abstract

In this paper we deal with the economic design of an $\overline{X}$ control chart used to monitor a quality characteristic whose observations fit to a first order autoregressive model. The Duncan cost model is used to select the control chart parameters, namely the sample size (*n*), the sampling interval (*h*) and the control limit coefficient (*k*), that lead to the optimal monitoring cost. We found that the autocorrelation has an adverse effect on the chart's power, on the false alarm risk and on the cost. It also increases *n* and *h* and decreases *k*. To counteract this undesired effect we considered setting up the subgroups using non-sequential observations. It is shown that this sampling strategy significantly reduces the monitoring cost.

Key-words: *Autocorrelation; First Order Autoregressive Model; Economic Design; Control Chart; Statistical Process Control.*

Introduction

Due to its inherent simplicity the $\overline{X}$ control chart is often selected to monitor the mean stability of a quality characteristic. The underlying assumption in Statistical Process Control (SPC) is that the observations from the process are independent and identically distributed (i.i.d.). However, in

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many applications the dynamics of the process induce serial correlation in observations closely spaced in time. The application of the $\overline{X}$ control chart for the monitoring of autocorrelated processes has been considered by several authors (Vasilopoulos & Stamboulis, 1978; Runger & Wilemain, 1995/1996; Gilbert et al, 1997; Sun & Xu, 2004; Zhang, 2006; Costa & Claro, 2008) who were primarily concerned with searching the chart properties and assessing its efficiency (typically expressed by the average run length, or the ARL).

When an $\overline{X}$ control chart is used to monitor a process, three parameters should be determined: the sample size, *n*, the sampling interval, *h*, and the control limits coefficients, *k*. The selection of these three parameters is usually called the design of the control chart. Comprehensive literature reviews for several different cost models and applications of various control charts are available in the literature (Montgomery, 1980; Vance, 1983; Ho & Case, 1994).

Given its widespread acceptance as a realistic model, we will adopt the methodology proposed by Duncan (Duncan, 1956) to represent the objective function which minimizes the average cost of monitoring processes subjected to a single assignable cause. Duncan's work was the cornerstone for much of the research in this area. Research studies conducted using the Duncan's model in connection with a number of Shewhart control charts for independent and cross-correlated processes are described next.

The Duncan's single cause cost model was embellished with Taguchi loss function in order to incorporate losses that result from inherent variability (Alexander et al, 1995). Although Duncan applied a penalty cost for operating out of control, there was no recommendation to the approach by which this cost could be obtained or quantified. This was considered and evaluated by the authors in their model. From the sensitivity analysis they found that *n* increases and *h* decreases to steady state values as the frequency of process shifts decreases. The rate of convergence to the steady state depends on the cost of searching for an assignable cause. The higher this cost, the slower the rate of convergence. In addition, they indicated that *n* and *h* have to be adjusted based on the size of the process shift that is investigated. For a small process shift, larger values of *n* and *h* are required; however for large shifts, they recommended a small value for *n* and *h* to be used.

The Weibull distribution was used to model the occurrence of assignable causes for the economic design of $\overline{X}-R$ charts (Costa & Rahim,

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2000). The authors modified the Duncan cost model by assuming a non-uniform decreasing sampling scheme to incorporate the effects of process deterioration and a two-step search procedure to determine the economically optimal design parameters. From the sensitivity analysis, they found that there is a significant cost penalty when using a uniform sampling scheme versus a non-uniform sampling scheme. In addition, when the scale parameter increases and the shape parameter decreases, h decreases whereas *n* and the penalty cost increase. Finally, they found that the optimal economic design is very sensitive to the value of the shape parameter.

The economic model of $\overline{X}$ chart for cross-correlated data was developed by Chou et al (2001). Duncan's single cause model was used as the objective function. The correlation model given in Yang & Hancock (1990) was applied to derive the error probabilities of the control chart. From their analysis, they found that highly positive correlated data results in a smaller *n*, *h* and *k*. Also, the power of the chart decreases as the value of the correlation coefficient increases. On the other hand, highly negative correlated data result in smaller *n* and *k*; however, *h* is not significantly affected.

Considering the Duncan cost model, Engin (2004) developed an application for the use of economic statistical $\overline{X}$ chart design in the textile yarn industry. Searching for the optimum *n*, *h* and *k* the author considered the power of the control chart to be at least 0.95 and the penalty-cost as minimal as possible.

Costa & De Magalhães (2005) presented a model for the economic design of a two-stage control chart based on both a performance variable (*X*) and a correlated, less expensive to measure surrogate variable (*Y*). The assumption of an exponential distribution to describe the length of time the process remains in control allowed the application of the Markov chain approach to obtain the chart's properties. The economic design considered by the authors was derived and properly adapted from the Duncan's model and the results confirmed that the two-stage model is better than the one-stage model in terms of the expected net income.

The literature dealing with the economic design of $\overline{X}$ chart is very rich and the same is true regarding to the number of papers dealing with the monitoring of autocorrelated processes, especially with the first order autoregressive model (Montgomery & Mastrangelo, 1991; Wardell et al, 1994;

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Runger & Willemain, 1996). Based on that, it seems worthwhile to consider the economic design of $\overline{X}$ charts for autocorrelated processes. In this paper, we study the economic design of the $\overline{X}$ chart when the process observations follow a first order autoregressive (AR1) model.

The Process Model and the $\overline{X}$ Contol Chart

Throughout this paper we assume that the observations of the quality characteristic to be monitored fit to a First Order Autoregressive AR(1) model, frequently employed in previous researches (Montgomery & Mastrangelo, 1991; Wardell et al, 1994; Runger & Willemain, 1996).

The process observations X_t can be written as:

$$X_t - \mu = \phi\left(X_{t-1} - \mu\right) + \varepsilon_t\,, \quad t=1, 2, 3,..., T \tag{1}$$

and the process variance is given by:

$$\sigma_X^2 = \frac{\sigma_\varepsilon^2}{1-\phi^2} \tag{2}$$

We additionally consider that the AR(1) model is accurate and the process autocorrelation is positive, ranging from low to moderately high levels $\left(0 < \phi \leq 0.75\right)$, believed to be prevalent in control charts applications.

The design of the $\overline{X}$ chart for the AR(1) process follows the concepts of the classical methodology applied to independent data, with centerline at μ_0 and control limits (*CL*) set at $\mu_0 \pm k\sigma_{\overline{X}}$; however, taking the autocorrelation into account to determine the sample standard deviation.

The type I error probability (α) of the $\overline{X}$ control chart is:

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$$\alpha = \Pr\left[Z > \frac{UCL - \mu_0}{\sigma_{\overline{X}}}\right] + \Pr\left[Z < \frac{LCL - \mu_0}{\sigma_{\overline{X}}}\right] \tag{3}$$

According to Alwan & Radson (1992), the standard deviation of a sample average taken from any stationary process, when the subgroups are “practically” independent, is:

$$\sigma_{\overline{X}} = \sqrt{\frac{\sigma_X^2}{n} + \frac{2}{n}\sum_{j=1}^{n-1}\left(1 - \frac{j}{n}\right)\gamma_j} \tag{4}$$

where:

σ_X^2 is the process variance

n is the sample size

γ_j is the autocovariance coefficient at lag *j*

The expression (4) can be rewritten as:

$$\sigma_{\overline{X}} = \frac{\sigma_X}{\sqrt{n}\,\Psi} \tag{5}$$

where:

$\Psi = \left[1 + \frac{2}{n}\sum_{j=1}^{n-1}(n-j)\rho_j\right]^{-1/2}$ and, for an AR(1) process, the autocorrelation coefficient at lag j is $\rho_j = \frac{\gamma_j}{\sigma_X^2} = \phi^j$.

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Assuming that $\mu_0 = 0$

$$\alpha = \Pr\left[\,|Z| > k \mid Z \sim \mathrm{N}(0,1)\,\right] \tag{6}$$

where *Z* is the standard normal variate and *k* is the coefficient of the control limits.

When the process is out-of-control $\mu = \mu_1 = \mu_0 + \delta\sigma_{\bar{X}}$. The type II error probability (β) of the $\overline{X}$ control chart is:

$$\beta = \Pr\left[\,LCL \le \overline{X} \le UCL \mid \mu \ne \mu_0\,\right] \tag{7}$$

where

LCL is the lower control limit

UCL is the upper control limit

μ is the mean of the process characteristic

μ_0 is the mean of the process characteristic when the process is in control

μ_1 is the mean of the process characteristic when the process is off-target

and the power of the chart is:

$$\mathrm{p} = 1 - \beta \tag{8}$$

that can be rewritten as:

$$\mathrm{p} = \Pr\left[Z < -k + \delta\sqrt{n}\,\Psi\right] + \Pr\left[Z < -k - \delta\sqrt{n}\,\Psi\right] \tag{9}$$

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The influence of the autocorrelation on the ARL is shown in Table 1.

Table 1 - Influence of the autocorrelation on the ARL, AR(1) model, n=4, ARL0=370.4

δ	$\phi = 0$	$\phi = 0.25$	$\phi = 0.50$	$\phi = 0.75$
0.25	157.6	192.3	226.7	256.7
0.50	45.1	66.1	93.1	123.7
0.75	15.5	25.1	39.5	58.4
1.00	6.5	11.0	18.5	29.4
1.25	3.4	5.6	9.6	15.9
1.50	2.1	3.3	5.5	9.3
1.75	1.5	2.2	3.5	5.8
2.00	1.2	1.6	2.4	3.9

It can be clearly seen that when the data autocorrelation increases the control chart's ability to detect process mean shifts is reduced.

Brief Review of the Duncan's Single-Assignable Cause Cost Model

In his pioneer article, Duncan proposed the consideration of economic factors in the design of a control chart (Duncan, 1956). The components of Duncan's cost model are:

a) the cost of an out-of-control condition.

b) the cost of false alarms.

c) the cost of finding an assignable cause.

d) the cost of sampling, inspection, evaluation and charting.

Duncan assumes that the process starts in-control and, at a random time, the process mean is subjected to a step shift. During the search for the assignable cause, the process is allowed to continue in operation. Duncan considered the production cycle depicted in Figure 1.

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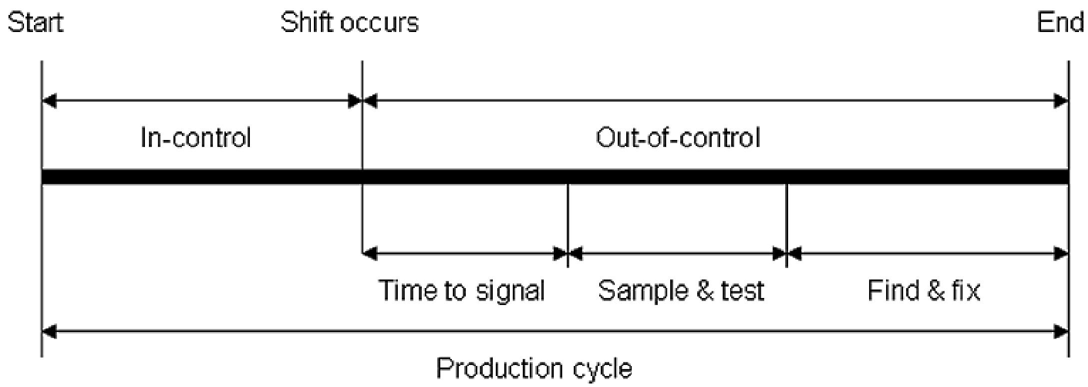

Figure 1 – The production cycle in the Duncan's model

When the average cycle length is determined the cost components can then be converted to a "per hour of operation" basis. The four components of average cycle length are as follows:

a) Assuming that the process begins in the in-control state, the time interval that the process remains in control is an exponential random variable with a mean of $1/\lambda$, which is the average process in-control time.

b) When the shift occurs, the process mean shifts, and the probability that this out-of-control condition will be detected on any subsequent sample is $1-\beta$ or the power of the chart. Thus, the expected number of subgroups taken before the detection of the process mean shift is $1/(1-\beta)$. The average time of occurrence within an interval between the j^{th} and the $(j+1)^{th}$ subgroups, given an occurrence of the assignable cause in the interval between these subgroups, is:

$$\tau = \frac{1-(1+\lambda h)e^{-\lambda h}}{\lambda\left(1-e^{-\lambda h}\right)} \tag{10}$$

Noting that the number of samples required to produce an out-of-

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control signal, given that the process is actually out of-control, is a geometric random variable with mean $1/(1-\beta)$, it can be concluded that the expected length of the out-of-control period is $h/(1-\beta)-\tau$.

c) The average sampling and testing time for each sample is a constant g proportional to the sample size n, so that the delay in plotting a subgroup point on the $\overline{X}$ chart is gn.

d) The time required to identify and fix the assignable cause following the signal is a constant D.

Therefore, the expected length of a cycle, denoted by *E*(*T*), is:

$$E(T)=\frac{1}{\lambda}+\frac{h}{1-\beta}-\tau+gn+D \tag{11}$$

The expected net income per cycle is:

$$E(C)=V_0\frac{1}{\lambda}+V_1\left(\frac{h}{1-\beta}-\tau+gn+D\right)-a_3-\frac{a_3' e^{-\lambda h}}{1-e^{-\lambda h}}-(a_1+a_2 n)\frac{E(t)}{h} \tag{12}$$

where V_0 is the net income per hour of operation in the in-control state, V_1 is the net income per hour of operation in the out-of-control state; a_1 and a_2 are respectively the fixed and variable sampling cost, a_3 is the cost of finding an assignable cause and a'_3 is the cost of investigating a false alarm.

The expected net income per hour is:

$$E(A)=\frac{E(C)}{E(T)}=V_0-E(L) \tag{13}$$

where:

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$$E(L) = \frac{a_1 + a_2 n}{h} + \frac{a_4\left[h/(1-\beta) - \tau + gn + D\right] + a_3 + a_3^{'}\alpha e^{-\lambda h}/\left(1 - e^{-\lambda h}\right)}{1/\lambda + h/(1-\beta) - \tau + gn + D} \tag{14}$$

The hourly penalty cost associated with production in the out-of-control state is a_4 and the quantity *E*(*L*) is called the penalty-cost and it is a function of the control chart parameters *n*, *k* and *h*. Obviously, the expected net income per hour reaches its maximum when *E*(*L*) is minimum.

The Influence of the Autocorrelation on the Optimal Cost

A *FORTRAN* program was coded for the minimization of the cost model described in expression (14). The program calculates the optimal control limit coefficient (*k*) and sampling frequency (*h*) for several values of *n* and computes the value of the corresponding α risk, chart's power and the cost function. To assess the effect of the serial dependency on the loss cost we borrowed the cost and model parameters found in Montgomery (2004), chapter 9, example 9-5, whose data is reproduced in Table 2, for the reader's convenience.

Table 2 - Cost and model parameters

a_1	a_2	a_3	$a_3^{'}$	a_4	λ	δ	g	D
1.00	.10	25.00	50.00	100.0	0.05	2.0	0.0167	1.0

(source: Montgomery, 2004, chapter 9, example 9-5)

In spite of the fact that the program calculates the cost for several samples sizes, we summarized in Table 3 only the combination of parameters that results in the optimal cost for processes where the autocorrelation level (ϕ) ranges from 0 to 0.75 in increments of 0.25.

The strategies commonly adopted so that the subgroups be "practically" independent are: (*i*) sufficiently large interval between subgroups, (*ii*) sufficiently large sample sizes and (*iii*) both (*i*) and (*ii*) combined. Runger and Willemain (1995) recommended selecting the sample size and/or the between-subgroups sampling interval in order to reduce the lag one autocorrelation of the subgroup means to no more than 0.10. Taking into account that the production rates in

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the modern industrial settings are substantially high (say, typically over 60 units per hour) the sample size and sampling interval equivalent to the minimum cost, see Table 3, are more than enough to mitigate the serial dependency to $\rho_1 \leq 0.10$ for AR(1) processes with $0 < \phi \leq 0.75$

Table 3 - Control chart parameters for the minimum monitoring cost of AR(1) processes (unconstrained design)

ϕ	n	optimum k	optimum h (hours)	alpha	power	optimum cost
0.00	5	2.99	.76	.0028	.9308	10.38
0.25	7	2.88	.80	.0040	.9169	10.89
0.50	9	2.68	.83	.0074	.8583	11.71
0.75	13	2.43	.87	.0151	.7682	13.32

It can be noted that when the autocorrelation increases, the power of the chart decreases and the sample size (*n*), the sampling interval (*h*), the false alarm risk (α) and the minimum cost all increase.

According to Woodall (1986) the economic method of designing control charts has several weaknesses. For example, the excessively large number of false alarms may introduce extra variability into the process through overadjustment and destroy confidence in the monitoring procedure. To overcome this issue, a statistical constraint of $\alpha \leq 0.0027$ was added to the economic design and the effects are shown in Table 4.

Table 4 - Control chart parameters for the minimum monitoring cost of AR(1) processes (constrained design: $\alpha \leq 0.0027$)

ϕ	minimum n	optimum k	optimum h (hours)	alpha	power	optimum cost
0.00	5	3.06	.82	.0022	.9210	10.38
0.25	7	3.05	.85	.0023	.8877	10.91
0.50	11	3.02	.94	.0025	.8566	11.82
0.75	19	3.00	1.00	.0027	.7620	13.96

The inclusion of the statistical constraint led to larger samples sizes and slightly higher cost; the control limit coefficients (*k*) and the sampling interval increased and the power of the chart was reduced.

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As it can be confirmed from Tables (3) and (4) the positive autocorrelation has a negative effect on the monitoring cost as well as on the chart's efficiency.

A Sampling Strategy to Partially Offset the Influence of the Autocorrelation on the Optimal Cost

The autocorrelation on the process observations has a large impact on control charts designed under the independence assumption. A typical effect is to increase the rate of false alarms (Lu & Reynolds Jr, 1999). Widening the control limits to encompass the systematic non-random behavior resulting from the positive correlation reduces the number of false alarms; however a special cause becomes not readily discernible, and consequently the monitoring cost increases. A proposed alternative to partially offset this effect is to attenuate the serial dependency by skipping items within the subgroup. To assess the efficiency of this strategy, we vary the gap between the subgroup observations, from *j*= 1 to *j*= {2, 3 and 4}. To exemplify, the observations in a subgroup of size 4 are shown in Table 5.

Table 5 - Observations included in a subgroup of size 4 for j={1,2, 3 and 4}

j	observations included
1	$X_i, X_{i+1}, X_{i+2}, X_{i+3}$
2	$X_i, X_{i+2}, X_{i+4}, X_{i+6}$
3	$X_i, X_{i+3}, X_{i+6}, X_{i+9}$
4	$X_i, X_{i+4}, X_{i+8}, X_{i+12}$

The efficiency of the control chart improves significantly when non-sequential observations are used to set up the subgroups, see Table 6.

We additionally computed the optimal cost and the cost reduction percentage (in comparison with the optimal cost when *j*=1) for processes with

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low, moderate and moderately high autocorrelation level assuming the model/cost factors given in Table 2 and the $(\alpha \le 0.0027)$ constraint on the false alarm rate, see Table 7.

If on the one hand, this sampling strategy leads to longer times to set up the sample; on the other hand it can be a helpful approach to reduce the monitoring cost and increase the power of the chart, see Figures 2 and 3.

Table 6 - Influence of the autocorrelation on the ARL, AR(1) model, *n*=4, j={1,2,3 and 4} and ARL_0=370.4 (unconstrained design)

ϕ	*gap*	δ							
		0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
0.00	*j*=1	157.6	45.1	15.5	6.5	3.4	2.1	1.5	1.2
0.25	*j*=1	192.3	66.1	25.1	11.0	5.6	3.3	2.2	1.6
	j=2	164.2	48.7	17.0	7.2	3.7	2.2	1.6	1.3
	j=3	158.3	45.8	15.8	6.7	3.5	2.1	1.5	1.2
	j=4	157.6	45.1	15.5	6.5	3.4	2.1	1.5	1.2
0.50	*j*=1	226.7	93.1	39.5	18.5	9.6	5.5	3.5	2.4
	j=2	192.3	66.1	25.1	11.0	5.6	3.3	2.2	1.6
	j=3	173.9	54.3	19.5	8.4	4.2	2.5	1.7	1.4
	j=4	164.2	48.7	17.0	7.2	3.7	2.2	1.6	1.3
0.75	*j*=1	256.7	123.7	58.5	29.4	15.9	9.3	5.8	3.9
	j=2	234.8	100.7	43.9	20.9	11.0	6.3	4.0	2.7
	j=3	216.5	84.3	34.6	15.8	8.2	4.7	3.0	2.1
	j=4	201.8	72.8	28.5	12.7	6.5	3.7	2.4	1.8

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Table 7 – Optimum cost and cost reduction percentage for AR(1) processes using different gapping strategies for the subgroup observations (constrained design: $\alpha \leq 0.0027$)

ϕ	lag (j)	n	optimum k	optimum h(hours)	alpha	power	optimum cost	cost reduction
0.25	1	7	3.05	0.85	0.0023	0.8877	10.91	
	2	6	3.05	0.85	0.0023	0.9453	10.48	3.94%
	3	5	3.07	0.79	0.0021	0.9110	10.41	4.58%
	4	5	3.06	0.82	0.0022	0.9119	10.39	4.76%
0.50	1	11	3.02	0.94	0.0025	0.8566	11.82	
	2	7	3.05	0.85	0.0023	0.8877	10.91	7.70%
	3	6	3.05	0.85	0.0023	0.9139	10.61	10.23%
	4	6	3.05	0.85	0.0023	0.9453	10.48	11.33%
0.75	1	19	3.00	1.00	0.0027	0.7620	13.96	
	2	13	3.01	0.97	0.0026	0.8583	12.17	12.80%
	3	11	3.02	0.94	0.0025	0.8920	11.46	17.90%
	4	8	3.04	0.88	0.0024	0.8839	11.12	20.34%

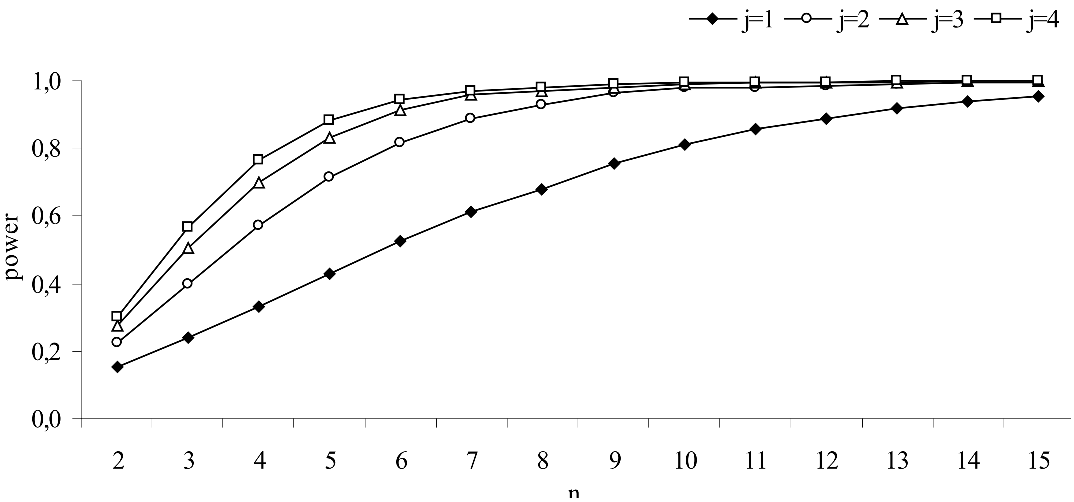

Figure 2 – Monitoring cost of an AR(1) process ($\phi = 0.50$) with $2 \leq n \leq 15$ and $1 \leq j \leq 4$

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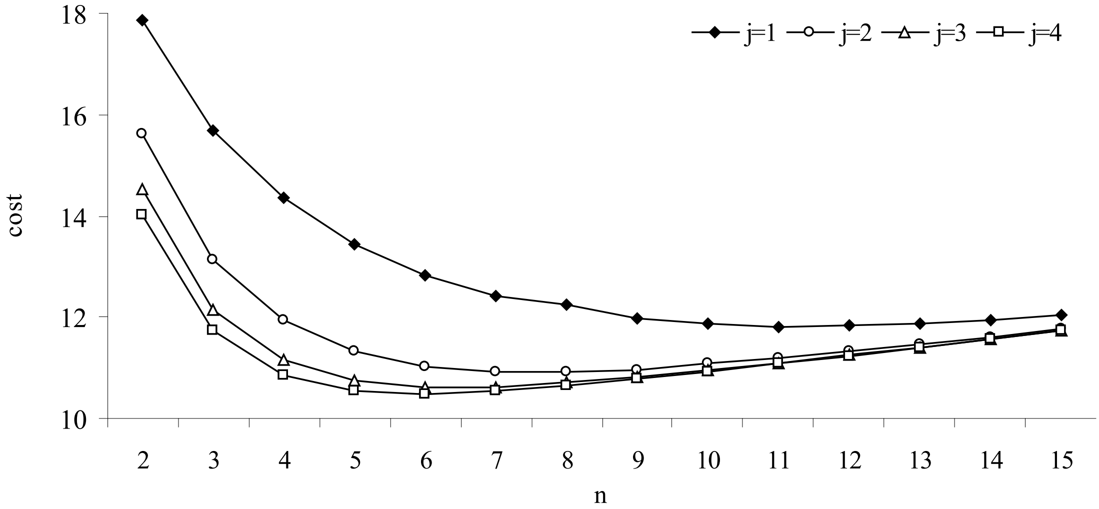

Figure 3 – Power of the chart of an AR(1) process $(\phi = 0.50)$ with $2 \leq n \leq 15$ and $1 \leq j \leq 4$

Conclusion

Most standard control charting schemes for SPC are based on the assumption that measurements of the product quality variable are independent. However, positive autocorrelation at low lags is commonplace in many industrial processes where, given the advances in sensor technologies, observations are closely spaced in time leading to within-subgroup serial correlation with undesirable effects on the performance of control charts and the optimal monitoring cost. Aiming to partially offset the influence of the autocorrelation, we propose a sampling strategy where non-sequential observations are used to set up the subgroups. In the present research, where the process observations were assumed to fit to an AR(1) process, the major findings are the following:

- The positive autocorrelation decreases the power of the chart, increases the false alarm risk (α) and the minimum cost. It also increases the sample size (*n*) and the sampling interval (*h*) and decreases the coefficient of the control limits (*k*).

- Skipping observations within the subgroup attenuates the serial dependency contributing to improve the chart's power and to reduce the monitoring cost.

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Nomenclature

n	sample size
h	sampling interval
k	coefficient of the control limits
Z	standard normal variate
$\overline{X}$	sample mean
μ	mean of the process characteristic
μ_0	mean of the process characteristic when the process is in control
μ_1	mean of the process characteristic when the process is off-target
σ_X	process standard deviation
$\sigma_{\overline{X}}$	sample means standard deviation
α	type I error probability of the control chart
β	type II error probability of the control chart
ϕ	coefficient of the AR(1) autoregressive model

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γ_j	autocovariance coefficient at lag *j*
ρ_j	autocorrelation coefficient at lag *j*
ε_t	AR(1) model errors, independent and identically distributed random variables with zero mean and variance σ_ε^2
p	power of the control chart
UCL	upper control limit
LCL	lower control limit
a_1	fixed sampling cost
a_2	variable sampling cost
a_3	cost of finding an assignable cause
a_4	hourly penalty cost of operating out-of-control
a'_3	cost of investigating a false-alarm
λ	reciprocal of the average process in-control time
δ	magnitude of the process mean step shift
g	average sampling, inspecting, evaluating and plotting time for each sample
D	time required to find the assignable cause
V_0	net income per hour of operation in the in-control state
V_1	net income per hour of operation in the out-of-control state
E(*T*)	expected length of a production cycle
E(*C*)	expected net income per cycle
E(*A*)	expected net income per hour
E(*L*)	expected loss per hour or the loss-cost

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